The Interstellar Medium 2010 Q4

Lecture 9: Shock waves

- Principles & examples
- Jump conditions
- J-type & C-type shocks
- Single point explosion in a uniform medium
- Supernova remnant evolution

Date	Topic	Book chapter
27-04-2010	Motivation, outline, Overview of ISM	1
	Part 1: Regions of ionized gas	
29-04-2010	Pure hydrogen nebulae	2.1.1 (atoms), 2.1.4
06-05-2010	Nebulae with heavier elements	7.1 - 7.3
11-05-2010	Heating & cooling of H^+ regions	2.2–2.4, 3.1-3.2
18-05-2010	Diagnostics of H^+ regions	7.4-7.5
20-05-2010	X-ray nebulae, novae, supernovae, AGN	12
	Part 2: Regions of neutral gas	
25-05-2010	Observational probes of neutral gas	8
27-05-2010	Thermal balance, Galactic distribution	2.5-2.6, 3.3-3.4
01-06-2010	Shock waves	11
03-06-2010	Three-phase model	9 (not 9.9-9.11)
	Part 3: Regions of molecular gas	
08-06-2010	Interstellar dust	5, 6, 13 (not 6.4-6.7)
10-06-2010	Molecular spectra	2.1.1 - 2.1.3, 8.7
15-06-2010	Molecular clouds	3.5-3.11, 4
17-06-2010	Star formation	10 (not 10.4-10.7)
22-06-2010	The extragalactic ISM	

Lecture 8: Heating and cooling of atomic gas

Heating mechanisms

p.i. carbon; p.e. dust & PAH; cosmic rays; X-rays

Cooling mechanisms

C^+ 158 µm; other fine structure lines; Lyα

Thermal balance & the two-phase ISM

multiple solutions co-exist in pressure equilibrium

Large-scale distribution of atomic gas

maps: flare & warp; tangent point; *l*-V diagram; rotation curve

Small scale features

21 cm ↔ 100 µm ↔ A_V ; HVCs as local DLA analogs

Homework: Tielens Chapter 11

11.1: introduction

shocks are ubiquitous

11.2: J-shocks

fast, sudden, hot, dissociative

11.3: C-shocks

slow, smooth, warm, magnetic, non-dissociative

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Lecture 9: Shock waves

Principles & examples

Jump conditions

J-type & C-type shocks

Single point explosion in a uniform medium

Supernova remnant evolution

Shock waves

Shock: pressure-driven compressive disturbance traveling faster than the local signal speed

Produces an *irreversible change* in the state of the fluid

Literature:

Landau & Lifschitz, *Fluid Mechanics*, chapter IV
Dyson & Williams 1997, Chapters 6 & 7
Tielens, Chapter 11

Sound speed and Mach number

Sound speed: $c_s^2 = dP / d\rho$

Take EoS $P = K\rho^\gamma$

$\gamma = 5/3$ for adiabatic fluid

$\gamma = 1$ for isothermal fluid

Adiabatic flow: $c_s \propto \rho^{1/3}$

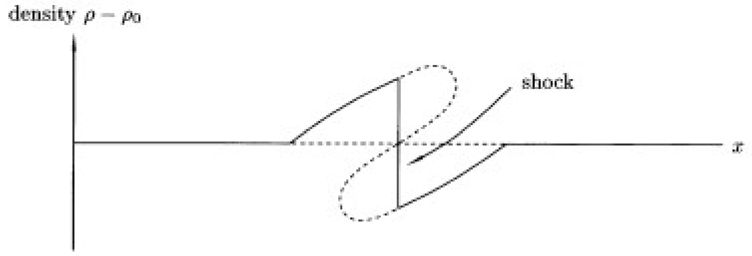
sound speed is larger in denser gas

For isothermal gas in ISM

$c_s = (kT/m)^{0.5} \approx 1$ km/s (very small!)

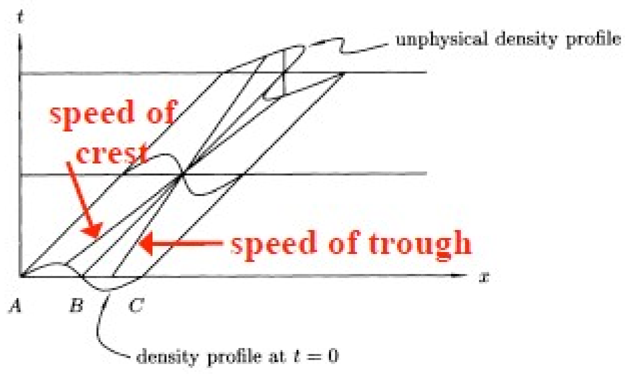
Mach number: $M = V / c_s$

Steepening of non-linear acoustic wave



Speed is larger in dense gas

- wave crest travels faster than trough
- crest catches up with troughs
- waveform steepens
- steepening halted by viscous forces
- development of a shock



Signal speeds in the ISM

In the absence of magnetic fields, information travels at the *sound speed*

$M < 1$: subsonic

$M > 1$: supersonic $\rightarrow$ shocks

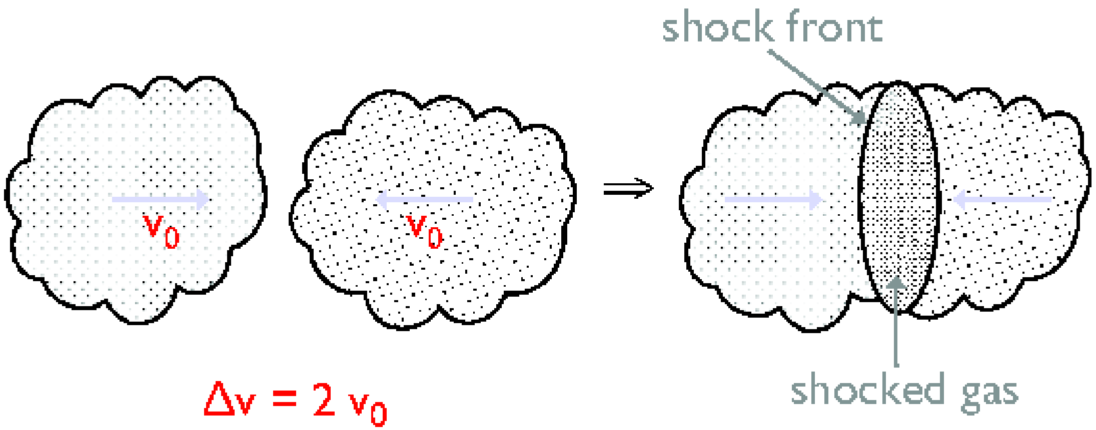
If magnetic field present, disturbances travel along the field lines at the *Alfvén speed*

$v_A^2 = B^2 / 4\pi\rho$

Interstellar field strengths: empirical law

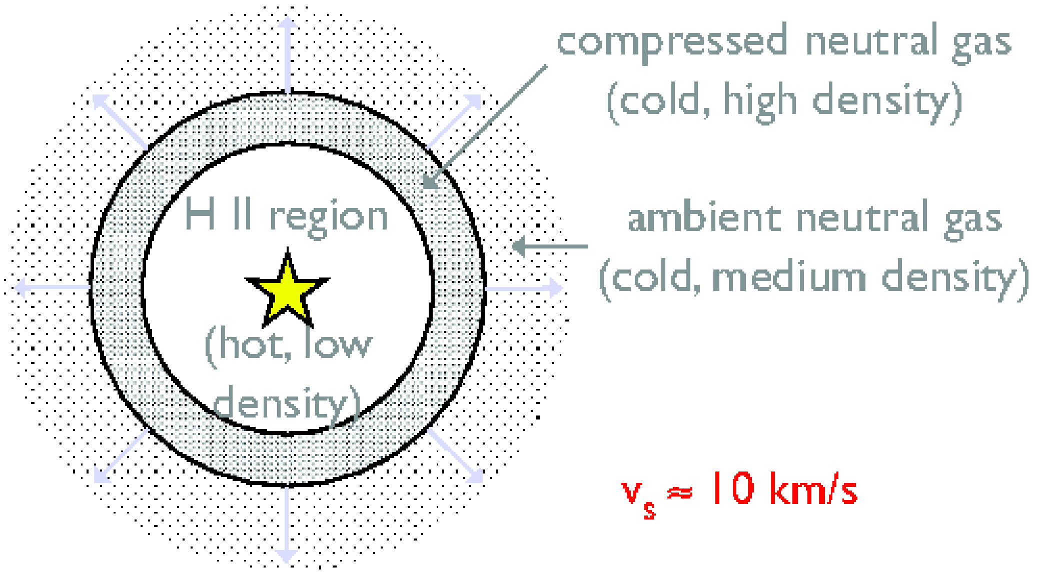
$B = 1\ \mu G \sqrt{n_H}$ for $10 < n_H < 10^6\ cm^{-3}$

Example 1: Cloud-cloud collisions



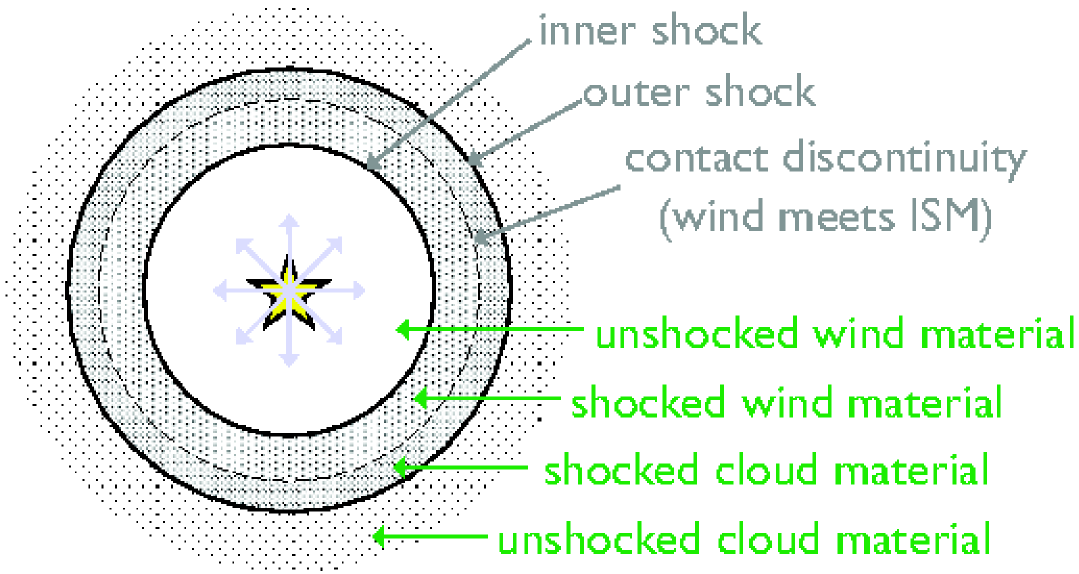
$v_0 = 5 - 10$ km/s; $v_S \approx v_0$

Example 2: Expansion of an H^+ region

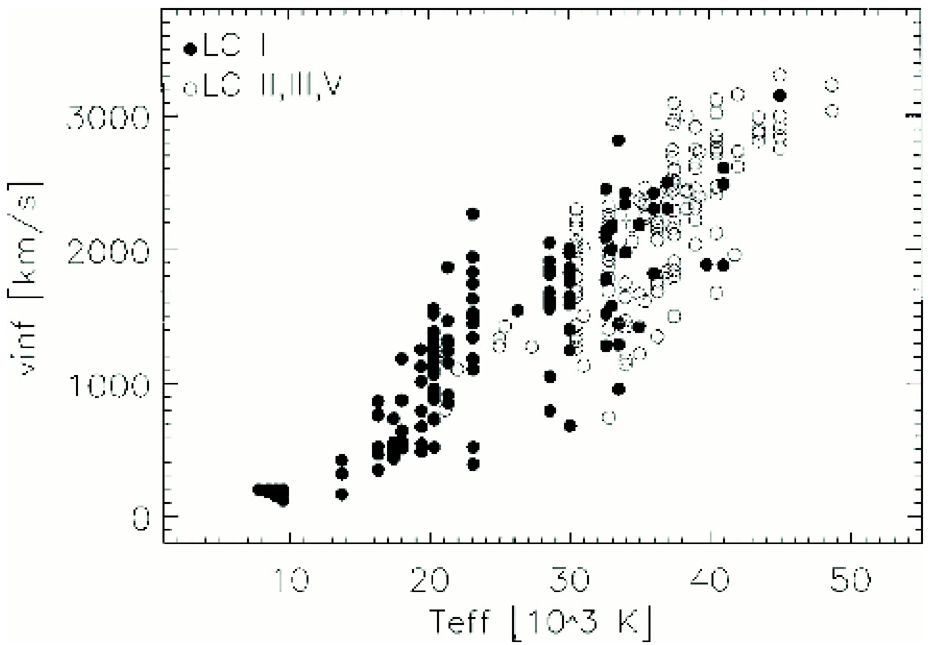


(only one phase in the evolution of an H^+ region)

Example 3: Fast stellar wind



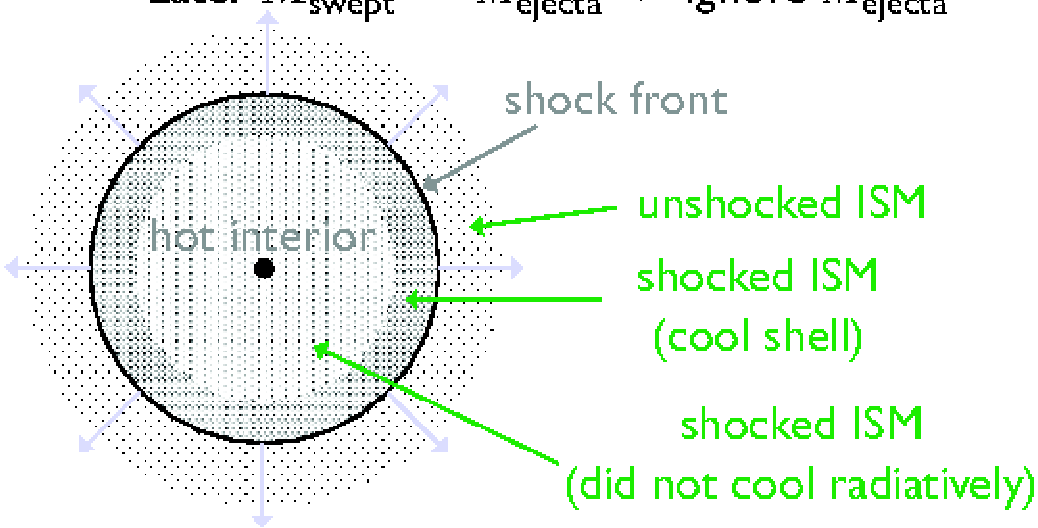
Speeds of stellar winds



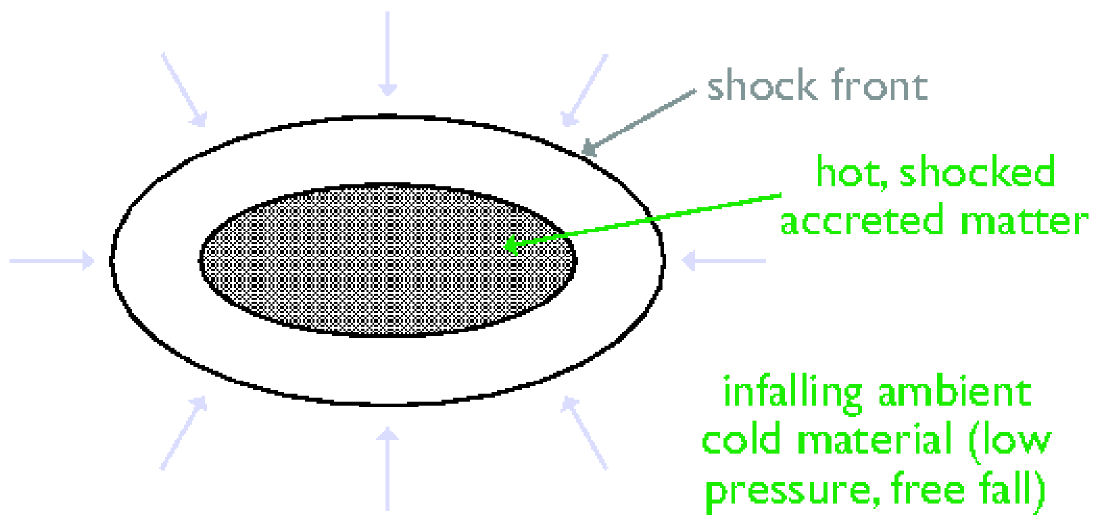
Example 4: Supernova blast wave

Early: $M_{\text{swept}} < M_{\text{ejecta}} \Rightarrow$ like stellar wind

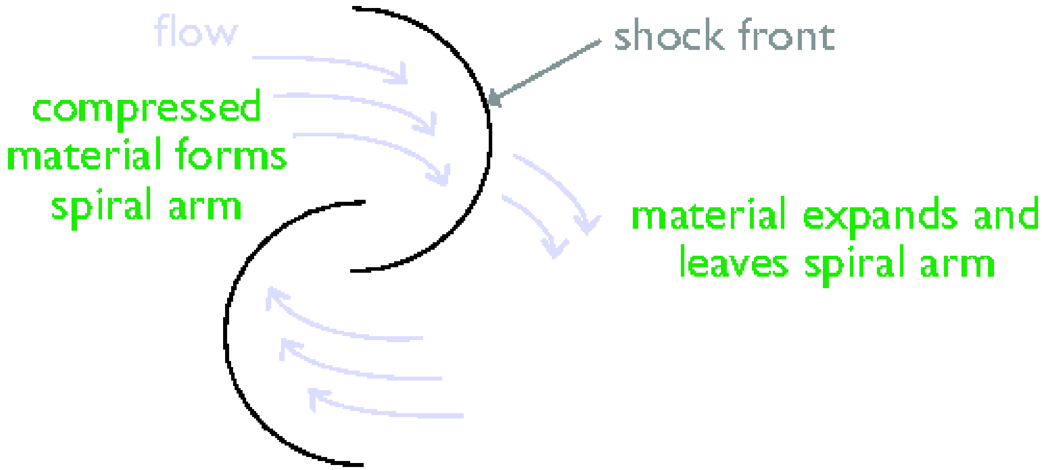
Late: $M_{\text{swept}} >> M_{\text{ejecta}} \Rightarrow$ ignore M_{ejecta}



Example 5: Star formation – infall & outflow



Example 6: Spiral shocks in Galactic disk



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Jump conditions for shocks

Adopt reference frame where shock is *stationary*

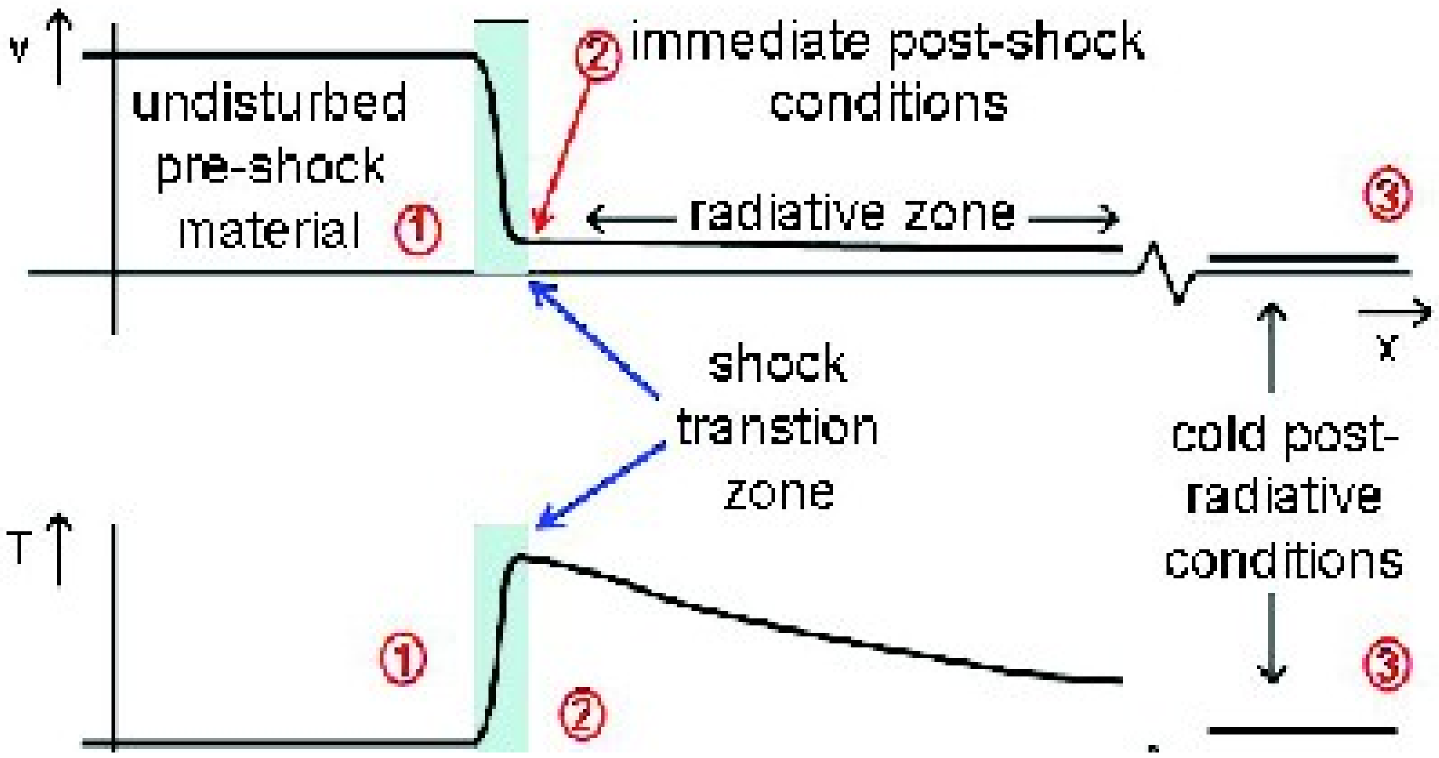
Consider *plane-parallel* shock

- conditions depend only on distance x from front

Neglect viscosity, except in transition zone:

- large velocity gradient
- viscous dissipation
- transform bulk kinetic energy into heat
- irreversible change: entropy increases

Velocity & temperature profiles (in shock frame)



More jump conditions

Thickness of shock front ≤ mean free path of particles

always << thickness of radiative zone

need collisions for radiative cooling

Regard thickness as infinitely small

discontinuity

Need to find physical conditions at

(2) immediately behind shock

or (3) in post-radiative zone

given those at

(1) pre-shock

and the shock velocity

Mass conservation

General case:

$$\partial\rho / \partial t + \nabla\cdot(\rho\mathbf{v}) = 0$$

For a steady plane-parallel shock:

$$\partial/\partial t = \partial/\partial y = \partial/\partial z = 0$$

$$\rightarrow \partial/\partial x\ (\rho v_x) = 0$$

Integrate across the shock:

$$(\rho v_x)_1 = (\rho v_x)_2$$

Momentum conservation

$\rho\, D\mathbf{v} / Dt = -\boldsymbol{\nabla}\, (P + B^2 / 8\pi) + 1/4\pi\, (\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{B} - \rho\, \boldsymbol{\nabla}\Phi$

or using $D / Dt = \partial/\partial t + v\cdot\boldsymbol{\nabla}$:

$\rho v_x\, \partial v_x/\partial x = -\partial/\partial x\, (P + B^2 / 8\pi) + (B_x / 4\pi)\, \partial B_x / \partial x - \rho\, \partial\Phi/\partial x$

which re-writes into

$$\partial/\partial x\, (\rho v_x{}^2 + P + (B^2 - B_x{}^2)/8\pi) = -\rho\, \partial/\Phi/\partial x$$

and assuming no change in the gravitational potential Φ

$(\rho v_x{}^2 + P + (B^2 - B_x{}^2)/8\pi)_1 = (\rho v_x{}^2 + P + (B^2 - B_x{}^2)/8\pi)_2$

Energy conservation

$$D / Dt\,(\tfrac{1}{2}\rho v^2 + U + B^2 / 8\pi) =$$

$$\partial/\partial x_i\,(-P\,v_i - v_i\,B^2 / 8\pi + v_i B_i B_k / 4\pi) - \rho v_i\,\partial\Phi/\partial x_i$$

$$- \nabla\cdot(-\kappa\Delta T) + (\Gamma - \Lambda)$$

(heat conduction / heating / cooling)

U = internal energy per unit volume = $p / (\gamma-1)$

Use same procedure as before:

$$\tfrac{1}{2}\rho v_x v^2 + U v_x + P v_x + v_x (B_y{}^2 + B_z{}^2)/8\pi) - B_x B_y v_y/4\pi -$$

$$B_x B_z v_z/4\pi - \kappa dT/dx = \int_1^2 (-\rho v_x \Phi + \Gamma - \Lambda)$$

which is ≈ 0 if 1 and 2 are close

Magnetic field

The electric field vanishes in the fluid frame

The Maxwell equation then reduces to

$$\partial B / \partial t = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + (c^2 / 4\pi\sigma)\, \boldsymbol{\nabla}^2\mathbf{B}$$

(flux freezing / field decay)

The last term describes *diffusion* of the magnetic field

vanishes if the conductivity $\boldsymbol{\sigma} \rightarrow \infty$ (ideal MHD)

Often-used simplifying assumptions

Planar shock: $v_y = v_z = 0$

No heat conduction: $\kappa \nabla T = 0$

No net heating or cooling: $\int_1^2 (\Gamma - \Lambda) = 0$

No gravitational force: $\int_1^2 \rho v_x \Phi \, dx = 0$

Internal energy density $U = P / (\gamma\text{-}1)$ with $\gamma = c_P / c_V$

Magnetic field perpendicular to flow: $B_x = 0$

may choose $B_y = 0$

Ideal MHD: $\partial/\partial x \, (v_x B_y - v_y B_x) = \partial/\partial x \, (v_z B_x - v_x B_z) = 0$

The Rankine – Hugoniot equations

Follow general practice

write $v_x = u_1$ ($= v_S$)

Mass conservation:

$$\rho_1 u_1 = \rho_2 u_2$$

Momentum conservation:

$$\rho_1 u_1^2 + p_1 + B_1^2 / 8\pi = \rho_2 u_2^2 + p_2 + B_2^2 / 8\pi$$

Energy conservation:

$$\tfrac{1}{2}\rho_1 u_1^3 + \gamma/(\gamma-1)\, u_1 p_1 + u_1 B_1^2 / 8\pi =$$

$$\tfrac{1}{2}\rho_2 u_2^3 + \gamma/(\gamma-1)\, u_2 p_2 + u_2 B_2^2 / 8\pi$$

Flux conservation:

$$u_1 B_1 = u_2 B_2$$

William
Rankine
1820 – 1872

Solving the Rankine – Hugoniot equations

The jump conditions are 4 equations in 4 unknowns:

ρ_2, u_2, P_2, B_2

Define $x = u_1 / u_2 = \rho_2 / \rho_1 = B_2 / B_1$

Now we have two equations in P_2 and x:

$$\rho_1 u_1^2 + P_1 + B_1^2 / 8\pi = \rho_1 u_1^2 / x + P_2 + x^2 B_2^2 / 8\pi$$

and

$$\tfrac{1}{2}\rho_1 u_1^3 + \gamma / (\gamma - 1)\, u_1 P_1 + u_1 B_1^2 / 8\pi =$$

$$\tfrac{1}{2}\rho_1 u_1^3 / x^2 + \gamma / (\gamma - 1)\, u_1 p_2 / x + x\, u_1 B_1^2 / 8\pi$$

Define "magnetic" Mach number: $M_m^2 = u_1^2 / v_{ms}^2$

where $v_{ms}^2 = \gamma P_1 / \rho_1 + B_1^2 / 4\pi\rho_1$

Solving the Rankine – Hugoniot equations (2)

The trivial solution 1 = 2 always exists

A second solution exists for $M > 1$

Strong shock ($M >> 1$):

$x \rightarrow (\gamma+1) / (\gamma-1)$

$= 4$ for $\gamma = 5/3$

and

$T_2 = 2 (\gamma-1) / (\gamma+1)^2 (\mu\, u_1^{\,2} / k)$

$= 3 / 16$ for $\gamma = 5/3$

so

$T_2 = 2300 (\mu / m_H) (u_1 / 10 \text{ km/s})^2$ K

Properties of strong shocks

The compression ratio $x = 4$

If $16\, B_1^{\ 2} / 8\pi \ll \rho_1\, u_1^{\ 2}$ and $P_1 \ll \rho_1\, u_1^{\ 2}$
then momentum conservation gives

$$P_2 \approx \tfrac{3}{4}\, \rho_1\, u_1^{\ 2} = \tfrac{3}{4} \text{ x ram pressure}$$

The magnetic pressure increases by 16: $B_2 / B_1 = 4$

The postshock flow is at constant pressure and subsonic

$$M_2 = u_2 / v_{ms2} = u_1 / 4\sqrt{(\gamma\, P_2 / \rho_2)} = 1/4\sqrt{(3/5 \times 16/3)} = 0.45$$

Radiative vs adiabatic shocks

So far considered shocks without radiative cooling
  "adiabatic shocks"
Bad terminology: shocks are abrupt and irreversible
  Adiabatic means here: no heat is removed

If postshock gas radiates line emission
  cools down in "radiative shock"

If temperature in region 3 is same as in region 1:
  "isothermal shock"
Bad terminology again:
  temperature always rises at shock front

Structure of "isothermal" shocks

Solve jump conditions with $T_3 = T_1$:

$\rho_3 >> \rho_1$

If $B = 0$: $\rho_3 / \rho_1 = M^2$

compression factor can be >> 4

If $B > 0$: $\rho_3 / \rho_1 = \sqrt{(8\pi \rho_1 u_1^2 / B_1^2)} = u_1 / v_{A1}$

$= 77 / b\ (u_1 / 100\ \text{km/s})$

where $B_1 = b\sqrt{n_1} \times 10^{-6}$ G

Typical compression factors $x \approx 100$

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J-type and C-type shocks

So far only considered single-fluid shocks

aka J-type shocks

Generally, ISM gas consists of 3 fluids

neutrals

ions

electrons

which can develop *different* temperatures and velocities

For $B > 0$, information travels by MHD waves

Perpendicular to B, the propagation speed is

$v_{ms} = \sqrt{(c_S^2 + v_A^2)}$

Magnetic precursors

Since $v_A \propto \sqrt{\rho}$ the Alfvén speed for decoupled ion-electron plasma can be much larger than if coupled

$$v_{A,ie} = B / \sqrt{(4\pi\rho_i)}$$

$$\approx 50 \text{ km/s } (10^4 \, n_i / n_H)^{0.5} \, (20 \, m_H \, n_i / \rho_i)^{0.5}$$

In many cases: $c_S < v_{A,n} < v_S < v_{A,ie}$

The ion-electron plasma sends information ahead of the disturbance and "informs" the pre-shock plasma that the compression is coming: magnetic precursor

The compression is now *subsonic* and the transition smooth and continuous: C-type shock

The ions then couple to the neutrals by collisions

Comparing J- and C-shocks

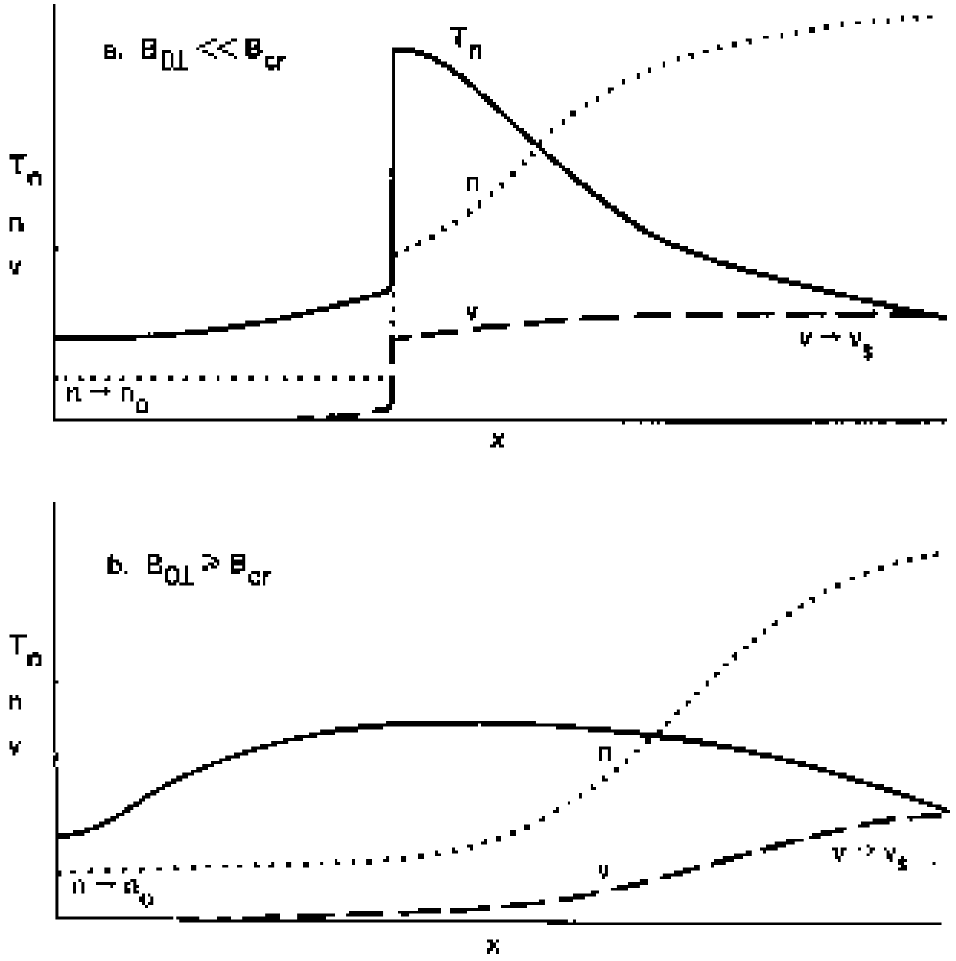
J-type ("jump") shocks: $v_s > 50$ km/s

- shock abrupt
- neutrals and ions tied into one fluid
- warm: $T = 40\ v_s^2$ [K; v in km s^{-1}]
- most radiation in *ultraviolet*

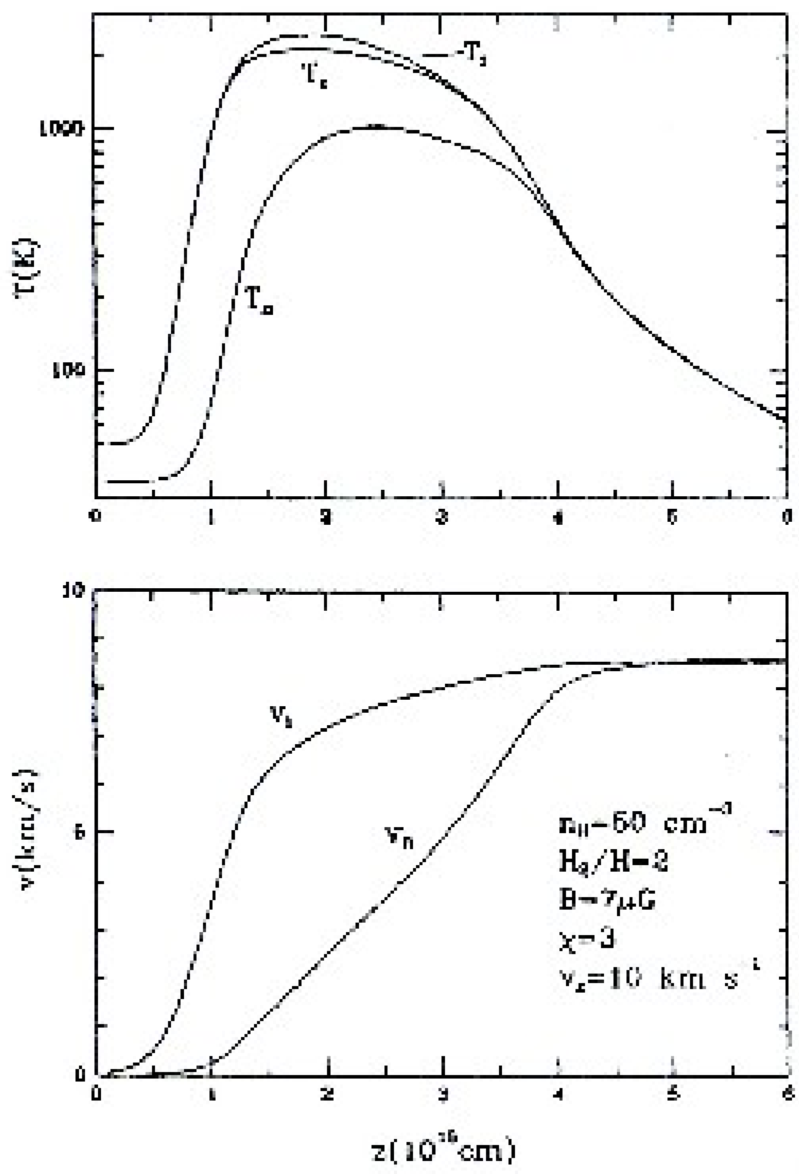
C-type ("continuous") shocks: $v_s < 50$ km/s

- gas variables (T, ρ, v) change gradually
- ions ahead of neutrals; drag modifies neutral flow
- $T_i \neq T_n$; both much lower than in J-shocks
- most radiation in *infrared*

Structure of J- and C-shocks



C-shock structure: Draine & Katz 1986



J-shock structure: Hollenbach & McKee 1989

Three regions:

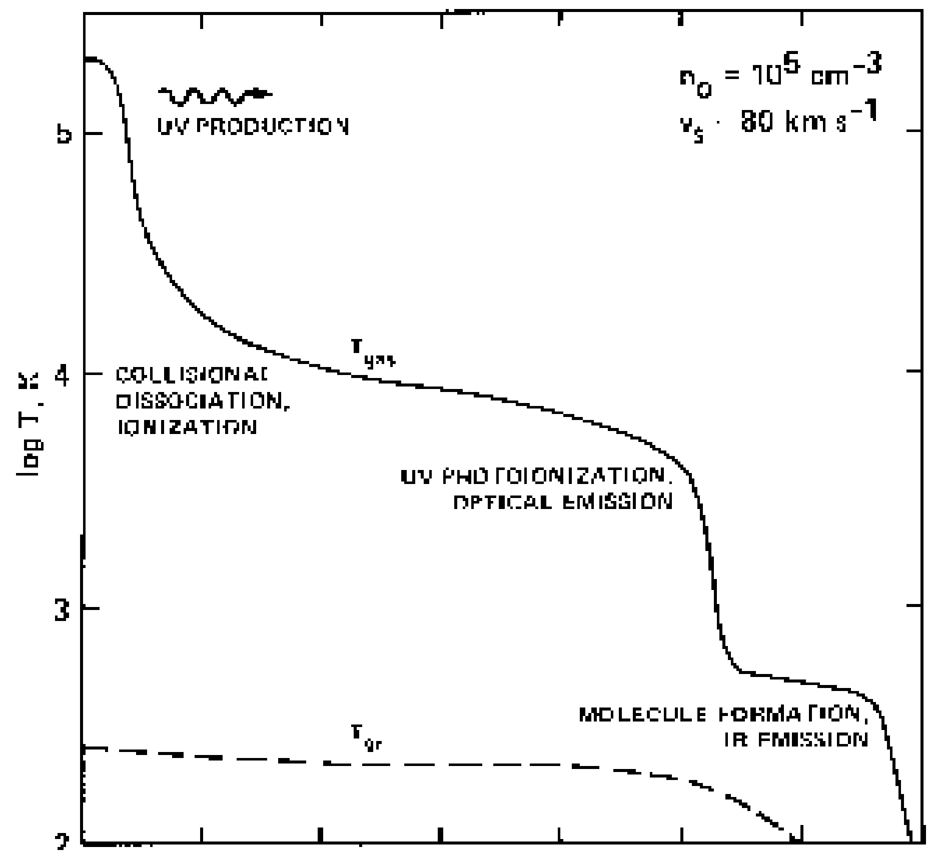
hot, UV emission

warm, Lyα absorption

cold, molecule formation

Weak gas-grain coupling:

$T_d << T_{kin}$



16 log N 22

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Single-point explosion

Idealized treatment due to Sedov (1959) and Taylor

See also Chevalier (1974)

Equally applicable to atomic bombs

Assumptions:

one-fluid treatment adequate (mean free path << size scale)

no radiative cooling

neglect viscosity & heat conduction

ambient pressure small

Characteristic variables:

size R

time t

velocity $v = R / t$

Self-similar solution

Characteristic quantities:

explosion energy *E*

ambient density ρ_0

time since explosion *t*

Evolution of *r*, *v* and *T* is described by universal functions of only one dimensionless parameter

$v\ (R,t) = R / t\ f_v(r/R)$

$\rho\ (R,t) = \rho_0\ f_\rho(r/R)$

$T\ (R,t) = mv^2 / k\ \ f_T(r/R)$

Dimensional analysis

Find dependence of R(t) on E, ρ_0 and t by writing

$R(t) = A\, t^{\alpha}\, E^{\beta}\, \rho_0^{\gamma}$

For *M*: $0 = 0 + \beta + \gamma$

For *L*: $1 = 0 + 2\beta - 3\gamma$

For *t*: $0 = \alpha - 2\beta + 0$

Result: $\alpha = 2/5,\ \beta = 1/5,\ \gamma = -1/5$

$R(t) = A\, (E\, t^2 / \rho_0)^{1/5} \propto t^{2/5}$

The numerical constant A

To estimate value of A, consider

expect $A \sim 1$

adiabatic case: $E_{tot} = E_{kin} + E_{th} = \text{constant}$

self-similar: $E_{kin} / E_{th} = \text{constant}$

Right behind shock ($v = ¾\, v_S$):

$E_{kin} = ½\, m\, (¾\, v_S)^2$

$E_{th} = 3/2\, k\, (3\, \mu\, v_S^2 / 16\, k)$

$E_{kin} / E_{th} = 1$

Detailed calculation of A

Assume most mass located just behind shock

OK from detailed solution

$E_{kin} \sim E_{th}$ for entire SNR

$E_{kin} = ½ M v_S^2$ with $M = 4\pi/3\, R^3 \rho_0$

Then:

$E_0 = 2E_{kin} = 4\pi/3\, R^3 \rho_0 v_S^2$

Writing $R = C t^\alpha$ we have $v_S = \alpha C t^{\alpha-1}$

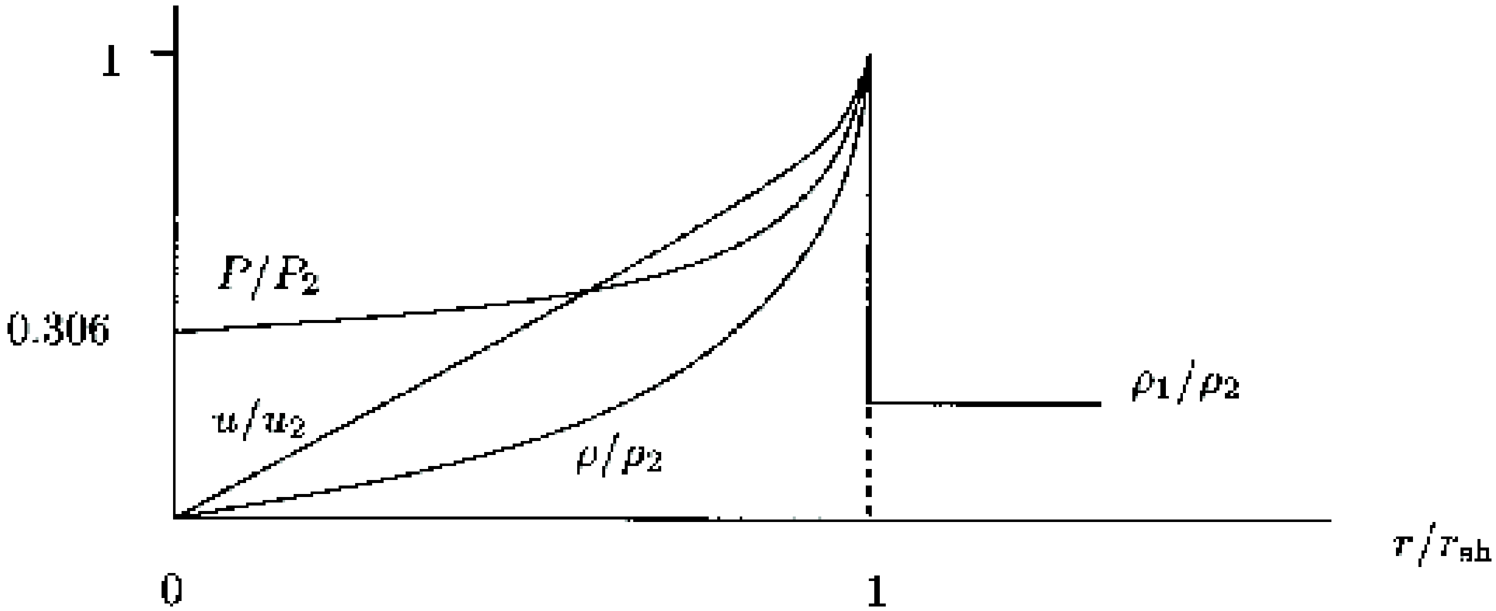
Then $E_0 = 4\pi/3\, \rho_0 C^5 \alpha^2 t^{5\alpha-2}$ = constant, so $\alpha = 2/5$

$C^5 = 3/4\pi\ (5/2)^2 (E_0/\rho_0)$

$C = 1.083\, (E_0/\rho_0)^{1/5}$ so $A = 1.083$

Exact solution for $\gamma = 5/3$: $A = 1.15169$

Sedov solution



50% of mass in outer 6% of radius

$T \propto P / \rho$ drops from center to edge

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Evolution of supernova remnants

Four phases:

- Early phase: $M_{swept} << M_{ejecta}$
- Sedov phase: $M_{swept} > M_{ejecta}$ and $t < t_{cool}$
- Radiative phase: $t \sim t_{cool}$
- Merging phase: $v_S < \Delta V_{ISM}$

The early phase

Condition: $M_{swept} << M_{ejecta}$

Free expansion: $R_S = v_S\, t$

Shock velocity ≈ ejecta velocity = $\sqrt{(2\, E_0 / M_{ejecta})}$ =

$= 10^4\, (E_0 / 10^{51}\, \mathrm{erg})^{0.5}\, (M_{ejecta} / M_{\odot})^{-0.5}$ km/s

Phase ends at $t = t_M$: $M_{swept} \approx M_{ejecta} = (4\pi\rho_0 /3)\, (v_0 t_M)^3$

→ $t_M = 190\, (M_{ejecta}/ M_{\odot})^{5/6}\, (E_0 / 10^{51}\, \mathrm{erg})^{-0.5}\, (n_H / \mathrm{cm}^{-3})^{-1/3}$ yr

The Sedov phase

Condition: $M_{swept} > M_{ejecta}$ but $t < t_{rad}$

Pressure-driven expansion:

cooling inefficient as long as $v_S > 250$ km/s

full ionization at $T \approx 10^6$ K

Self-similar solution:

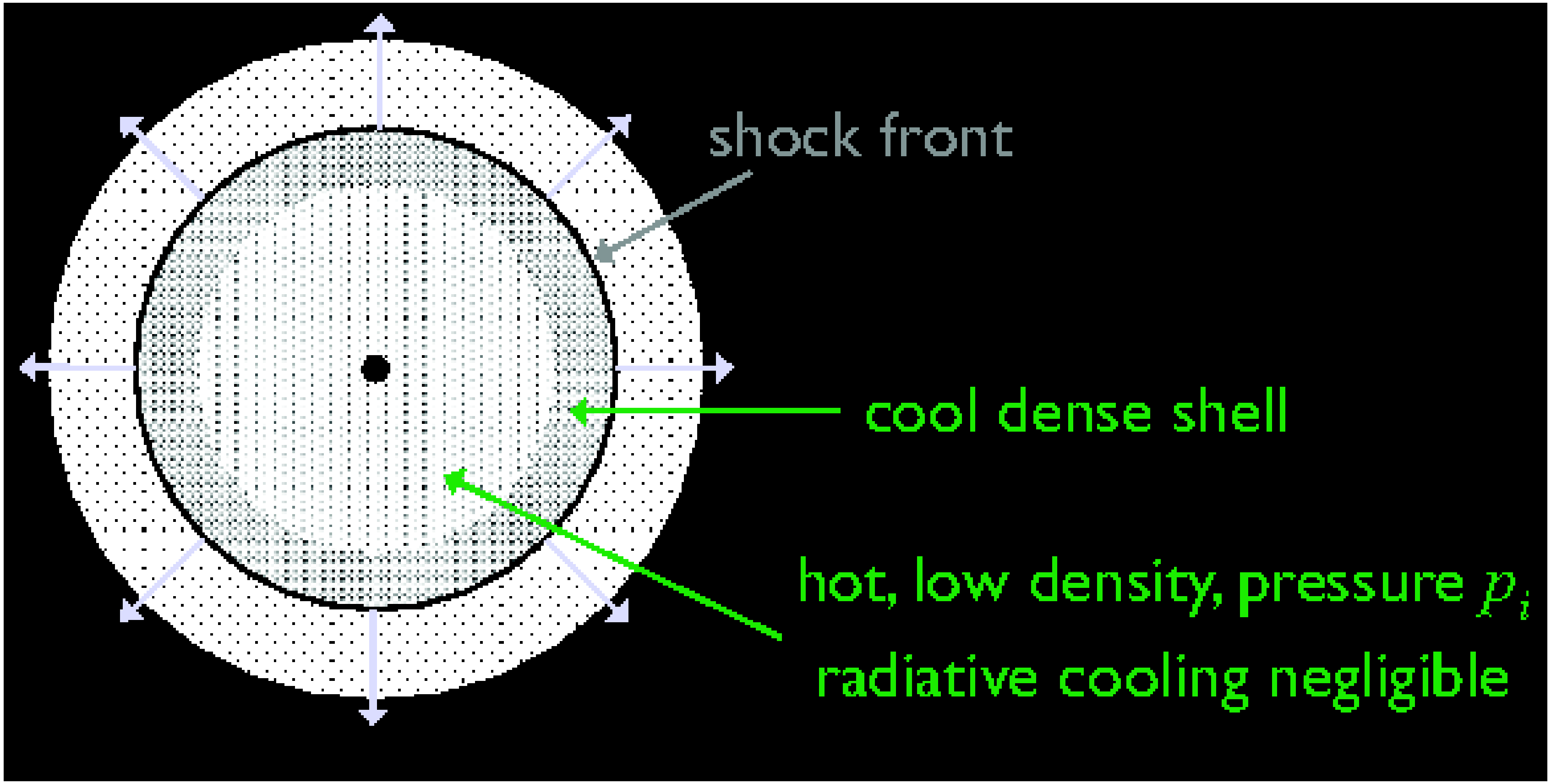
$R = 1.15167\ (E_0\ t^2 / \rho_0)^{1/5}$; $v_S = 2R / 5t$

Numerical calculation:

$R = 32\ (E_0 / 10^{51}\ \mathrm{erg})^{1/5}\ (n_H / \mathrm{cm}^{-3})^{-1/5}\ (t / 10^5\ \mathrm{yr})^{2/5}$ pc

$v_S = 120\ (E_0 / 10^{51}\ \mathrm{erg})^{1/5}\ (n_H / \mathrm{cm}^{-3})^{-1/5}\ (t / 10^5\ \mathrm{yr})^{-3/5}$ km/s

Cooling at end of Sedov phase



The end of the Sedov phase

To estimate t_{rad} note that

most cooling occurs just behind shock

high ρ, low T

Cooling rate: $\Lambda = 2 \times 10^{-25}\, n_H\, n_e\, (T / 10^7\ \mathrm{K})^{-2/3}$ erg s^{-1} cm^{-3}

Half of $E_{thermal}$ is radiated when:

$t_{rad} = 4.4 \times 10^4\, (E_0 / 10^{51}\ \mathrm{erg})^{2/9}\, (n_H / \mathrm{cm}^{-3})^{-5/9}$ yr

$v_S(t_{rad}) = 200\ (E_0 / 10^{51}\ \mathrm{erg})^{1/15}\, (n_H / \mathrm{cm}^{-3})^{2/15}$ km/s

$R_S(t_{rad}) = 23\ (E_0 / 10^{51}\ \mathrm{erg})^{13/45}\, (n_H / \mathrm{cm}^{-3})^{-19/45}$ pc

Note that shock velocity almost constant

The radiative phase

Cold dense shell slows down

by sweeping up ambient ISM

momentum is conserved

Naive snowplow: neglect P_i

$d(Mv) / dt = 0 \rightarrow M\, v \sim M_{cool}\, v_{cool}$

$R^3\, v = R^3_{cool}\, v_{cool} \rightarrow v \propto R^{-3}$

$v \propto R / t \rightarrow R \propto t^{1/4}$

The Oort snowplow

Include P_i:

$$d(Mv) / dt = P_i \, 4\pi R^2$$

P_i drops due to adiabatic expansion:

$$P_i = P_{cool} \, (R / R_{cool})^{-3\gamma}$$

Together, using $\gamma = 5/3$:

$$v \, 4\pi R^2 \rho_0 v + 4\pi/3 \, R^3 \rho_0 \, dv/dt = P_{cool} \, (R / R_{cool})^{-5} \, 4\pi R^2$$

So we have:

$$R = R_{cool} \, (t / t_{cool})^{2/7}$$

$$v_S = v_{S,\,cool} \, (t / t_{cool})^{-5/7}$$

The merging phase

The SNR fades away when

expansion velocity ≈ velocity dispersion ambient gas

Using the Oort snowplow:

$(t_{fade} / t_{cool})^{5/7} = v_{S,\ cool} / 10$ km/s = 200 / 10 = 20

So we have:

$t_{fade} = 2.9 \times 10^6 (E_0 / 10^{51}\ \mathrm{erg})^{0.32} (n_H / \mathrm{cm}^{-3})^{0.27}$ yr

$R_{fade} = 76 (E_0 / 10^{51}\ \mathrm{erg})^{0.32} (n_H / \mathrm{cm}^{-3})^{-0.16}$ pc

Summary of supernova remnant evolution

Early phase

$M_{swept} << M_{ejecta}$

Free expansion: $R_s = v_s\ t$

Sedov phase

$M_{swept} > M_{ejecta}$ & $t < t_{rad}$

energy conservation: $R \propto t^{2/5}$

Radiative "snowplow" phase

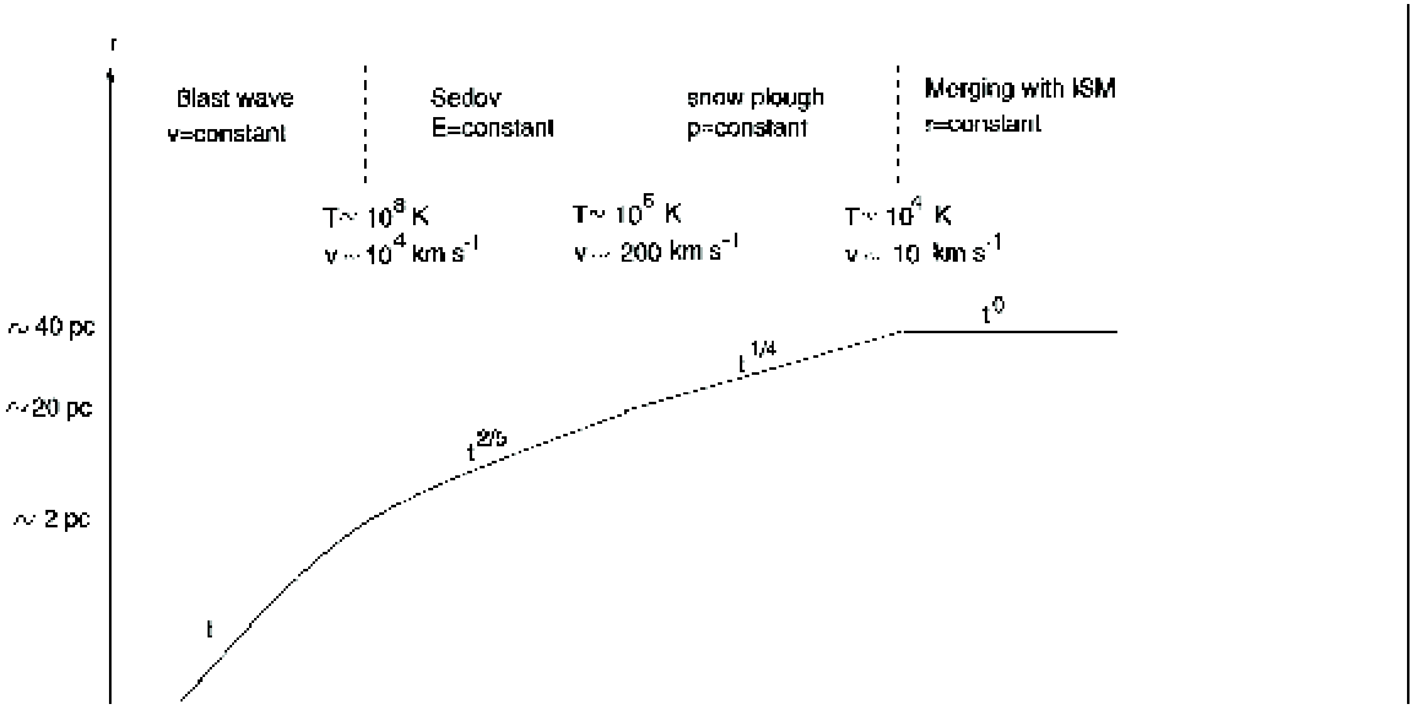
$t > t_{rad}$

momentum conservation: $R \propto t^{1/4}$ or $R \propto t^{2/7}$

Merging phase

v_s drops below ΔV of ambient ISM

Overview of supernova remnant evolution



200 yr 3 x 10^4 yr 3 x 10^5 yr

Leonid Ivanovitch Sedov 1907 – 1999

Devised similarity solution for blast waves

First chairman of USSR space exploration program

President of the International Astronautical Federation

Played leading role in Sputnik project